§ 18. Through any point S of a given surface four bifocal right lines may in general be drawn. Supposing the surface to be central, let a plane drawn through the centre, parallel to the plane which touches the surface at S, intersect any one of these right lines. Then the distance of the point of intersection from the point S will always be equal to the primary semiaxis of the surface.*

If through any point S of a given central surface a right line be drawn touching two other given surfaces confocal with it, and if this right line be intersected by a plane drawn through the centre parallel to the plane which touches the first surface at S, the distance of the point of intersection from the point S will be constant, wherever the point S is taken on the first surface. If this constant distance be called l, and the other denominations be the same as in the formula (7), the value of l will be given by that formula.†

Professor Mac Cullagh communicated the following note relative to the comparison of arcs of curves, particularly of plane and spherical conics.

The first Lemma given in my paper on the rectification of the conic sections (Transactions of the Royal Irish Academy, vol. xvi., p. 79) is obviously true for curves described on any given surface, provided the tangents drawn to these curves be *shortest lines* on the surface. The demonstration remains exactly the same; and the Lemma, in this general form, may be stated as follows.

Understanding a tangent to be a shortest line, and supposing two given curves E and F to be described on a given

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* Exam. Papers, An. 1838, p. xlvii., quest. 9.

† In the notes to the last mentioned work of M. Chasles, on the History of Methods in Geometry, will be found many theorems relative to surfaces of the second order. Among them are some of the theorems which are given in the present paper; but it is needless to specify these, as M. Chasles's work is so well known.

surface, let tangents drawn to the first curve at two points T, *t*, indefinitely near each other, meet the second curve in the points P, *p*. Then taking a fixed point A on the curve E, if we put *s* to denote (according to the position of this point with respect to T) the sum (or difference) of the arc AT and the tangent TP, and $s + ds$ to denote the sum (or difference) of the arc A*t* and the tangent *tp*, we shall have *ds* equal to the projection of the infinitesimal arc P*p* upon the tangent; that is, if a be the angle which the tangent TP makes with the curve F at the point P, we shall have *ds* equal to P*p* multiplied by the cosine of a.

Now through the points P, *p* conceive other tangents T′P, *t′p* to be drawn, touching the curve E in the points T′, *t′*; and let *s′* and *ds′* have for these tangents the same signification which *s* and *ds* have for the former tangents. Supposing the nature of the curve F to be such that it always bisects, either internally or externally, the angle made at the point P by the tangents TP and T′P, it is evident that $ds = \pm ds'$, and therefore either $s + s'$ or $s - s'$ is a constant quantity.

A simple example of this theorem is afforded by the plane and spherical conics. If the curves E and F be two confocal conics, either plane or spherical, and tangents TP, T′P be drawn to F from any point P of E (the tangents being of course right lines when the curves are plane, and arcs of great circles when they are spherical; in both cases shortest lines) it is well known that the angle TPT′ made by the tangents is always bisected by the conic E. The angle is bisected internally or externally according as the conics intersect or not. Hence we have the two following properties* of confocal conics:—

* The first of these properties was originally given for spherical conics by the Rev. Charles Graves, Fellow of Trinity College, in the "notes and additions" to his translation of M. Chasles's Memoirs on Cones and Spherical Conics,

1. When two confocal conics do not intersect, if one of them be touched in the points T, T′ by tangents drawn from any point P of the other, the sum of the tangents TP, T′P will exceed the convex arc TT′ lying between the points of contact, by a constant quantity.

2. When two confocal conics intersect in the point A, if one of them be touched in the points T, T′ by tangents drawn from any point P of the other, the difference between the tangents TP, T′P will be equal to the difference be-the arcs AT, AT′.

These properties give the readiest and most elegant solution of problems concerning the comparison of different arcs of a plane or spherical conic. Any arc being given on a conic, we may find another arc beginning from a given point, which shall differ from the given arc by a right line if the conic be plane, or by a circular arc if the conic be spherical.

DONATIONS.

Memoires de la Société Géologique de France. Tom. 5. Parts 1, 2. Presented by the Society.

The Tenth Annual Report of the Royal Cornwall Polytechnic Society. (1842.) Parts 1 and 2. Presented by the Society.

Bulletin de l'Academie Royale de Bruxelles, from 5th of November, 1842, to 8th of July, 1843.

p. 77 (Dublin, 1841). Mr. Graves obtained it as the reciprocal of the proposition, that when two spherical conics have the same directive circles, any tangent arc of the inner conic divides the outer one into two segments, each of which has a constant area. Both properties, with the general theorem relative to curves described on any surface and touched by shortest lines, were afterwards given in the University Calendar. See Exam. Papers, An. 1841, p. xli., quests. 3–6; An. 1842, p. lxxxiii., quests. 30–34. These two properties of conics were communicated, in October 1843, to the Academy of Sciences of Paris, by M. Chasles, who supposed them to be new. See the *Comptes rendus,* tom. xvii. p. 838.